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MATHEMATICAL MODELLING: LINKING MATHEMATICS, SCIENCE, AND THE ARTS IN THE PRIMARY CURRICULUM

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This paper presents one approach to incorporating interdisciplinary experiences in the primary school mathematics curriculum, namely, the creation of realistic mathematical modelling problems that draw on other disciplines for their contexts and data. The paper first considers the nature of modelling with complex systems and how such experiences differ from existing problem-solving activities in the primary mathematics curriculum. Principles for designing interdisciplinary modelling problems are then presented, with reference to two mathematical modelling problems, one based in the scientific domain and the other in the literary domain. Examples of the models children have created in solving these problems follow. Finally, a reflection on the differences in the diversity and sophistication of these models raises issues regarding the design of interdisciplinary modelling problems.

> Modelling and theory building lie at the intersection of art-science-mathematics. The history of model building in science conveys epistemological awareness of domain limitations. Arts imagine possibilities, science attempts to generate models to test possibilities, mathematics serves as the tool (Sriraman & Dahl, in press).

INTRODUCTION

Our world is increasingly governed by complex systems. Financial corporations, education and health systems, the World Wide Web, the human body, and our own families are all examples of complex systems. Complexity—the study of systems of interconnected components whose behaviour cannot be explained solely by the properties of their parts but from the behavior that arises from their interconnectedness—is a field that has led to significant scientific methodological advances (Sabelli, 2006).

In the 21st century, such systems are becoming increasingly important in the everyday lives of both children and adults. For all citizens, an appreciation and understanding of the world as interlocked complex systems is critical for making effective decisions about one's life as both an individual and as a community member (Bar-Yam, 2004; Davis & Sumara, 2006; Jacobson & Wilensky, 2006; Lesh, 2006).

Educational leaders from different walks of life are emphasizing the need to develop students' abilities to deal with complex systems for success beyond school. These abilities include: constructing, describing, explaining, manipulating, and predicting complex systems (such as sophisticated buying, leasing, and loan plans); working on multi-phase and multi-

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component projects in which planning, monitoring, and communicating are critical for success; and adapting rapidly to ever-evolving conceptual tools (or complex artifacts) and resources (English, 2002; Gainsburg, 2006; Lesh & Doerr, 2003). One approach to developing such abilities is through mathematical modelling, which is central to the study of complexity and to modern science. Meaningful inquiry, which involves cycles of model construction, evaluation, and revision, is fundamental to mathematical and scientific understanding and to the professional practice of mathematicians and scientists (Lesh & Zawojewski, 2007; Romberg, Carpenter, & Kwako, 2005).

Modelling is not just confined to mathematics and science, however. Other disciplines including economics, information systems, finance, medicine, and the arts have also contributed in large part to the powerful mathematical models we have in place for dealing with a range of complex systems (Steen, 2001; Lesh & Sriraman, 2005; Sriraman & Dahl, in press). Unfortunately, our mathematics curricula do not capitalize on the contributions of these external disciplines. A more interdisciplinary and unifying model-based approach to students' mathematics learning could go some way towards alleviating the well-known "one inch deep and one mile wide" problem in many of our curricula (Sabelli, 2006, p. 7; Sriraman & Dahl, in press; Sriraman & Steinthorsdottir, in press). We have limited research, however, on ways in which we might incorporate other disciplines within the primary mathematics curriculum. I offer one such approach here, namely, the creation of realistic mathematical modelling problems that draw on other disciplines for their contexts and data.

This paper first considers the nature of modelling with complex systems and how such experiences differ from existing problem-solving activities in the primary mathematics curriculum. Principles for designing interdisciplinary modelling problems are then presented, with reference to two mathematical modelling problems, one based in the scientific domain and the other in the literary domain. Examples of the models children have created in solving these problems follow. Finally, a reflection on the differences in the diversity and sophistication of these models raises issues regarding the design of interdisciplinary modelling problems.

THE POWER OF MODELLING

Modelling is increasingly recognized as providing students with a "sense of agency" in appreciating the potential of mathematics as a critical tool for analyzing important issues in their lives, their communities, and in society in general (Greer, Verschaffel, & Mukhopadhyay, in press). Indeed, new research is showing that modelling promotes students' understanding of a wide range of key mathematical and scientific concepts and "should be fostered at every age and grade…as a powerful way to accomplish learning with understanding in mathematics and science classrooms" (Romberg et al., 2005, p. 10). Students' development of potent models should be regarded as among the most significant goals of mathematics and science education (Lesh & Sriraman, 2005).

As Greer et al. point out, mathematical modelling has traditionally been reserved for the secondary and tertiary levels, with the assumption that primary school children are incapable of developing their own models and sense-making systems for dealing with complex situations. However, recent research (e.g., English, 2006; English & Watters, 2005) is showing that younger children can and should deal with situations that involve more than just simple counts and measures, and that entertain core ideas from other disciplines.

Modelling for the Primary Classroom

The terms, models and modelling, have been used variously in the literature, including in reference to solving word problems, conducting mathematical simulations, creating representations of problem situations (including constructing explanations of natural phenomena), and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp, 1999; English & Halford, 1995; Gravemeijer, 1999; Greer, 1997; Lesh & Doerr, 2003; Romberg et al., 2005; Van den Heuvel-Panhuizen, 2003).

The perspective adopted here is that models are "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (Doerr & English, 2003, p.112). From this perspective, modelling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the traditional school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal (Lesh & Zawojewski, 2007).

A focus on modelling differs from existing approaches to the teaching of mathematics in the primary classroom. First, *the quantities and operations that are needed to mathematize realistic situations often go beyond what is taught traditionally in school mathematics*. The types of quantities needed in realistic situations include accumulations, probabilities, frequencies, ranks, and vectors, while the operations needed include sorting, organizing, selecting, quantifying, weighting, and transforming large data sets (Doerr & English, 2001; English, 2006; Lesh, Zawojewski, & Carmona, 2003). As indicated next, modelling problems provide children with opportunities to generate these important constructs for themselves.

Second, *modelling problems offer richer learning experiences than the standard classroom word problems ("concept-then-word problem*," Hamilton, in press). In solving such word problems, children generally engage in a one- or two-step process of mapping problem information onto arithmetic quantities and operations. In most cases, the problem information has already been carefully mathematized for the children. Their goal is to unmask the mathematics by mapping the problem information in such a way as to produce an answer using familiar quantities and basic operations. These word problems constrain problem-solving contexts to those that often artificially house and highlight the relevant concept (Hamilton, in press). They thus preclude children from creating their own mathematical constructs out of necessity. Indeed, as Hamilton (in press) notes, there is little evidence to suggest that solving standard textbook problems leads to improved competencies in using mathematics to solve problems beyond the classroom.

In contrast, modelling provides opportunities for children to elicit their own mathematics as they work the problem. That is, the problems require children to make sense of the situation so that they can mathematize it themselves in ways that are meaningful to them. This involves a cyclic process of interpreting the problem information, selecting relevant quantities, identifying operations that may lead to new quantities, and creating meaningful representations (Lesh & Doerr, 2003).

Third, *mathematical modelling explicitly uses real-world contexts that elicit the creation of useful systems or models and draw upon several topic areas not only from mathematics but also from other disciplines.* For example, in the "*Creek Watch*" problem (Appendix A [1-4], children developed models to determine the water quality of their local creek. In doing so, they engaged with core ideas from the "Life and Living," "Science and Society," and "Systems, Resources, and Power" strands of their primary science and social sciences curricula. In the problem in Appendix B ("*Summer Reading*") children develop a model to determine a fair way to assign points to readers who enrol in a summer reading program. Children need to consider the number and variety of books read, their difficulty levels, the length of the books, and the quality of the readers' reports. "Experientially real" contexts such as children's local creek or community/school library provide a platform for the growth of their mathematization skills, thus enabling children to use mathematics as a "generative resource" in life beyond the classroom (Freudenthal, 1973; Stevens, 2000; Streefland, 1993).

Fourth, *modelling problems encourage the development of generalizable models*. In my research with elementary and middle school children, I have implemented sequences of modelling problems that encourage the creation of models that are applicable to a range of related situations (e.g., Doerr & English, 2003; Doerr & English, 2006; English & Watters, 2005). Children are initially presented with a problem that confronts them with the need to develop a model to describe, explain, or predict the behavior of a given system (a model-eliciting problem). Given that re-using and generalizing models are central activities in a modelling approach to learning mathematics and science, the children then work related problems that enable them to extend, explore, and refine those constructs developed in the initial problem (model-exploration and model-application problems). Because the children's final products embody the factors, relationships, and operations that they considered important, powerful insights can be gained into the children's mathematical and scientific thinking as they work the problem sequence.

Fifth, *modelling problems are designed for small-group work where members of the group act as a "local community of practice" solving a complex situation* (Lesh & Zawojewski, 2007). Numerous questions, issues, conflicts, revisions, and resolutions arise as the children develop, assess, and prepare to communicate their products to their peers. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team and other class members.

DESIGNING INTERDISCIPLINARY MODELLING PROBLEMS FOR THE PRIMARY SCHOOL

My research has involved working with teachers to design mathematical modelling problems that align themselves with the learning themes being implemented in the classroom. Such themes have included, among others, natural disasters, the local environment, classroom gardens, early colonisation, the gold-rush days, the Olympic and Commonwealth Games, book reading clubs, and class excursions to fun parks. The instructional design principles of Lesh and his colleagues (e.g., Lesh, Cramer, Doerr, Post, & Zawojewski, 2003, p. 43) are applied in designing these problems. These principles include the following:

1. The Personal Meaningfulness Principle

It is important that children can relate to and make sense of the complex system being presented in the problem. That is, the system should be one that reflects a real-life situation and that builds on children's existing knowledge and experiences. By designing problems that are integrated within the classroom's particular learning theme, the activities are less likely to be treated as "add-ons" in an already crowded curriculum. The modelling problems serve to not only enrich the problem-solving component of the mathematics curriculum but to also help children link their learning meaningfully across disciplines. For example, the Creek Watch problem incorporates core scientific and societal understandings such as *environments are dynamic and have living and non-living components that interact*, *living organisms depend on others and the environment for survival*; and *the activities of people can change the balance of nature*.

2. The Model Construction Principle

A modelling problem should require children to develop an explicit mathematical construction, description, explanation, or prediction of a meaningful complex system. The models children create should be mathematically significant. That is, they should focus on the underlying structural characteristics (key ideas and their relationships), rather than the surface features, of the system being addressed. For example, in the Summer Reading problem, as indicated later, children developed models that involved significant mathematisation processes such as assigning value points, using interval quantities, weighting selected factors, aggregating quantities, and applying informal measures of rate.

3. The Model Documentation Principle

Modelling problems should encourage children to externalise their thinking and reasoning as much as possible and in a variety of ways. The need to create representations such as lists, tables, graphs, diagrams, and drawings should be a feature of the problem. Furthermore, the models children construct need to involve more than a brief answer: descriptions and explanations of the steps children took in constructing their models should be included.

4. The Self-Assessment Principle

Modelling activities should provide children with sufficient criteria for determining whether their final model is an effective one and adequately meets the client's needs in dealing with the given complex system (and related systems). Such criteria also enable children to progressively assess and revise their creations as they work the problem. For example, in the Summer Reading problem, children are informed that the students who enrol in a reading program often read between ten and twenty books over the summer and that they can select books from any grade level. In developing their model for assigning points to each reader, children need to take into account the number and variety of books read, the difficulty level, the book length, and the quality of the written reports.

5. The Model Generalisation Principle

The models that children generate should be applicable to other related problem situations. As previously noted, by implementing a sequence of related modelling problems, children are provided with opportunities to extend, explore, and refine the models they developed in an initial problem (e.g., Doerr & English, 2003).

CHILDREN'S MODEL CONSTRUCTIONS FOR THE CREEK WATCH AND SUMMER READING PROBLEMS

This section addresses two longitudinal studies in which the Creek Watch and Summer Reading problems were implemented respectively. The models that the children created in solving each problem are presented.

In analysing the children's model developmental as they work these problems, I considered several aspects of their responses. These include the ways in which the children interpreted and re-interpreted the problem, the nature of the problem factors children chose to work with, the shifts in their thinking that led them to more sophisticated conceptual understanding, the mathematisation processes they applied to the given data including ways in which they quantified qualitative data, the types of data transformations they made through these processes, the representations they used in documenting and supporting their thinking, and the diversity in the final models they created.

Creek Watch Problem

Participants, Design, and Procedures

Four classes of third-grade children (7-8 year-olds) and their teachers from a suburban Brisbane state school participated in this longitudinal study, which spanned the children's third-, fourth-, and fifth-grade levels. The Creek Watch problem was implemented at the beginning of the fifth grade.

A teaching experiment involving multilevel collaboration (English, 2003; Lesh & Kelly, 2000) was employed. At the first level, children engage in mathematical modelling, as discussed previously. At the second and third levels, the classroom teacher works collaboratively with the researchers in designing and implementing the modelling problems. These problems also serve as challenging and thought-provoking experiences for the teachers as they explore the mathematical ideas being developed, consider appropriate implementation strategies, and promote learning communities within their classrooms. To facilitate the teachers' development, a number of workshops, meetings, and debriefing sessions were held throughout each year of the study.

At the beginning of each of the first two years of the study the children completed a number of preparatory activities prior to completing 2-3 comprehensive modelling problems during the remainder of the year. These preparatory activities included interpreting mathematical and scientific information presented in text and diagrammatic form; reading simple tables of data; collecting, analysing, and representing data; preparing written reports from data analysis; working collaboratively in group situations; and sharing end products with class peers by means of verbal and written reports.

For the Creek Watch problem, implemented at the beginning of the study's third year, the class teachers were given background reading on modelling and on the core scientific concepts embedded in the problem. Prior to implementing the problem, the teachers conducted a preparatory lesson addressing the main scientific ideas. During the first problem session, the children were presented with the following information:

(a) The *Creek Watch Activity* (see Appendix A[1]), which included background information on river ecology, a set of "readiness" questions, and the problem itself;

(b) A *Creek Watch "Fact Sheet*,*"* which provided information on water quality as determined by: (i) the presence of macro-invertebrates (diversity and sensitivity); (ii) the type and quantity of fish (native or exotic); (iii) the presence of algae and other plants including weeds; and (iv) the concentration or amounts of various chemicals in the creek. For example, the higher the level of nitrogen the poorer the quality of water; dissolved oxygen is most critical to water quality; high levels of turbidity are detrimental to the water quality; and most organisms are able to survive in water with a pH of between pH6.5 and pH9.0 (normal pH is 7.0).

(c) An explanation of the *Pollution Index* (see Appendix A[2]);

(d) A *Water Bugs Identification Chart* (see appendix A[3]); and

(e) *Moggill Creek Data Table* (see Appendix A[4]).

The children completed the problem activities in four sessions, each of 40-60 minutes duration over a period of 2.5 weeks. The children worked the problem in small groups (2-4 children per group), without direct instruction from the teacher or researchers. During the last session the children presented group reports to the class on the models they had created. In each classroom, 2-3 groups were videotaped and 2-4 groups were audiotaped as they worked the problem; all whole class reports were also videotaped. Each of the tapes was transcribed for analysis and all of the children's written artifacts were also examined.

Following their completion of this activity, the classes applied their models in exploring their local creek behind their school.

Children's Models for the Creek Watch Problem

A selection of the children's models follows. These models ranged from little or no application of mathematisation processes through to transformation of data using ranking and assigning of scores. Some models comprise inappropriate operations on the data, as can be seen in models 3 and 4.

Model 1: Selecting the healthiest site for each factor

The groups who developed this model listed all the problem factors one under the other (i.e., each of the macro-invertebrates, fish, weeds, and chemicals) and identified the healthiest site for each factor. For example, site A was selected as the best site for dragonfly nymphs, which are highly sensitive creatures, while site E was chosen as the cleanest with respect to each weed. The number of times each site was chosen was tallied and the site/s with the highest total was identified as the cleanest. This model did not include the calculation of a pollution index for each site, nor did model 2.

Model 2: Classifying each factor as "good" or "bad" for each site

A variation of model 1 entailed classifying each factor as "good" (√) or "bad" (x) for each site. The macro-invertebrates were first classified as "sensitive," "tolerant" and "very tolerant," with "sensitive" and "tolerant" considered good and the "very tolerant" as bad. For example, for dragonfly nymphs, sites A and C were rated as good while the remaining sites were rated as bad. A table was drawn for each set of factors across the five sites (i.e., a table for macro-invertebrates for sites A-E, a table for fish for sites A-E etc.) and each site awarded a tick or cross accordingly. The cleanest site/s had the greatest number of ticks across all factors.

Model 3: Determining the PI for the macro-invertebrates and a "chemical index" for each site

Although the majority of groups determined the pollution index for the macro-invertebrates for each site, there were variations in how they dealt with the remaining factors. Two groups calculated a "chemical index" for each site by adding the units of all the chemicals, ignoring the differences in the nature of the chemicals and their units. The two indexes were then totalled to determine the water quality of the sites. As one group explained in their final letter:

> Dear Jack and your group,
>
> We are writing to you to help you work out how to test and find out how polluted the water is. First you have to collect the data from the sites (animals, fish, chemicals etc) that you and your group have been to. Secondly you will have to write down the animals, chemicals etc. and you add the sensitivity number for the animals and for the chemicals. You add the numbers for different chemicals to find out how polluted the water is. Once you have added these up (animals) you will end up with the pollution index total and you will end up with a pollution index also for the chemicals. If the answer (animals) is quite high, then the water will be quite clean. If the answer (chemicals) is quite low then the water is quite dirty. When we worked it out we found that Site A was quite clean, site B was sort of dirty, and sort of clean. For site C it was also quite dirty and quite clean. Site D was very clean and site E was very clean.

The group did not consider the fish and weeds.

Model 4: Determining the PI for each site and quantifying the fish and weed data

A variation of model 3 involved a consideration of the PI for each site plus a consideration of the fish and weed data only. One group simply totalled the fish and weed data, ignoring the fact that some fish were not desirable. The fish and weed totals for each site were combined with the site's PI to give an overall score as an indication of the water quality (the site with the highest score was deemed the cleanest).

Model 5: Determining the PI for each site and describing impact of remaining factors

In this model only the PI was calculated, with the remaining factors considered individually for their respective impact on the water quality at each site. This impact was not quantified, however. For example, one group presented the following report, which stops short of indicating which site is the cleanest when the remaining factors are taken into account:

> Dear Jack Simpson,
>
> We heard about your problem and we would like to help solve your problem. First find the pollution index of the Moggill Creek by adding the points of the macro-invertebrates. The points we got were: site A- 51, site B- 35, site C- 17, site D- 30, site E- 49. After finding the pollution index of the area look at the other data. If an area has less bad fish (platy, guppy, swordtail) and more good fish (eel fish, black mangrove, carp, purple-spotted gudgeon) the area is good. With the macro-invertebrates and the fish there are weeds. The weeds are alligator weeds, Chinese elm, and camphor laurel. The less weeds an area has the less it is polluted. Also there are chemicals, good chemicals like dissolved oxygen, OK chemicals salinity (salt) and bad chemicals like turbidity, phosphorous, nitrogen, and pH (a good pH level is 7.0-7.05).

Model 6: Transforming the data through ranking

This model represented a more sophisticated version of model 2 and involved a ranking of the factors across the sites. This group of children constructed tables for each set of factors and ranked each site from 1 to 5 according to the desirability/non-desirability of the factor. For the macro-invertebrates, the children only considered selected examples and first recorded their sensitivity score. The children then identified the best site for each of these macro-invertebrates and ranked that site as 1. Each of the remaining sites was then ranked accordingly from 2 through 5. The site with the greatest number of 1s was chosen as the best site for macro-invertebrates.

A similar process was followed for the chemical analysis factors (see Figure 1). For dissolved oxygen, the sites were ranked from 1 (highest amount) to 5 (lowest amount). For turbidity and salinity, the sites were ranked from 1 (lowest amount) to 5 (highest amount). The remaining chemicals were ranked in the same way.

For the fish, only the exotic were considered, with each site ranked from 1 (least amount) to 5 (highest amount). Finally, the number of weeds at each site was aggregated, with site E clearly the best site with respect to lack of weeds (the children omitted their rankings from their weed table). The site with the greatest number of 1s was declared the cleanest site. The children explained their system in the following report, although their description of the ascending and descending ranking is not entirely clear.

> Dear Jack Simpson
>
> We found that Site E is the healthiest site of the creek. We did this by drawing graphs of macro-invertebrates, fish, weeds, and chemical analysis. Using a fact sheet we found out whether they are good or bad for the environment. Depending on whether they were good or bad, we ranked them. If they are good we ranked them in descending order but if they were bad we'd rank them in ascending order.

> For the dissolved oxygen we put a number for the one that has the most, which is site A and then for the turbity, salinity, phosphorous, nitoergen, and other notorgen we put the one that was the least because they are all bad. Then we counted up the one that has the most ones. It was site E. We did that to the other ones (factors) as well.
>
> Then we put a tally beside the number that was ranked first. The site with the most tallies is the most healthiest. Site E. We hope our guidelines help you.

INSERT FIGURE 1 ABOUT HERE

Model 7: Transforming the data by assigning scores

This model is a variation of models 6 and 1. In developing their model, the children suggested rating each site out of five for each of the factors (*You know how there's five sites. We'll give them a rating out of five for each of them, like, one, two, three, four.…we'll look at our data and if that is bad or not and we'll tally at the end, once we've gone through all of it, would be the most healthiest*). However, the children did not continue with this suggestion, rather, for each desired factor, they decided to award one point to the site that had the "best" of that factor. If the factor was not desirable, the site that had the least of that factor was given one point (*If it was bad we looked for the least and if it was good we looked for the most…we didn't really give any points for bad*). All points were then tallied and the site with greatest number of points was chosen as the cleanest. The children completed a bar graph showing the overall number of points awarded to each site.

The children's approaches to solving the Creek Watch problem are revisited in the discussion section. Consideration is now given to the study in which the Summer Reading problem was implemented and the models created by seventh-grade children in solving the problem.

Summer Reading Problem

Participants, Design, and Procedures

In this longitudinal study I worked with one class of children and their teachers from the fifth grade (9-10 years-old) through to the end of the seventh grade. The children attended a suburban, private co-educational P-12 college. As in the previous study, a teaching experiment involving multilevel collaboration was used.

In the first year of the study a number of preparatory modelling activities were implemented, followed by a model-eliciting problem and a model-exploration problem. In each of the second and third years an initial model-eliciting problem, a model-exploration problem, and two model-application problems were implemented. The problems involved interpreting and dealing with multiple tables of data; creating, using, modifying, and transforming quantities; exploring relationships and trends; and representing findings in visual and text forms. Also inherent in the problems were the key mathematical ideas of rate and proportion, and ranks and weighted ranks. The children had had no formal instruction on these core mathematical ideas and processes prior to commencing the problem activities.

The Summer Reading Problem (see Appendix B) was the final activity that the children completed in their seventh grade; the problem was a model-application activity where the children extended, explored, and refined constructs they had developed in the previous modelling problems. The problem required the children to develop a fair rating system to award points to students participating in a summer reading program. The children worked the problem in small groups during two 50-minute sessions. They then presented their models to the class in a third session, where they explained and justified the models they had developed and subsequently invited feedback from their peers. This group reporting was followed by a whole class discussion that compared the features of the mathematical models produced by the various groups.

As for the previous study, data sources included audiotapes and videotapes of the children's group work and classroom presentations. Field notes, children's work sheets, and final reports detailing their models and how they developed them were also important data sources. The tapes were transcribed and analysed for evidence of the mathematical understandings and mathematisation processes that the children used in building their models. To assist in the analysis, a modified version of Carmona's (2004) assessment tool for describing students' mathematical knowledge was used. The analysis addressed the nature of the problem factors that the children chose to consider, the operations they applied, the types of transformations they made through these operations, and the representations they used in documenting their final model.

Children's Model Creations

Table 1 displays the problem factors, mathematical operations, and representational formats used by each of five student groups in developing their models. The representational formats included the use of tables, text, lists, and formulae. In constructing their models, children chose to work with some or all of the problem factors, namely, number and variety of books read, reading level and length of the books, a student's grade level, and the quality of written reports. Children's operations on these factors included assigning value points, using interval quantities, using weighting, aggregating quantities, and using informal notions of rate. One group of children also imposed constraints on the use of their operations.

INSERT TABLE 1 ABOUT HERE

As can be seen in Table 1, the variety of books read was considered by only one group in creating a fair rating system while four out of the five groups took into account a student's grade level and a book's reading level. It is perhaps not surprising that the variety of books read presented a challenge—the notion of variety can be interpreted in different ways and its measure involves a consideration of several factors. As one group explained, "*With different variety of books—just say like if you read three different variety of books in subject or level or pages—just variety, so varieties include level, pages, and subject*." It is also interesting to note that all but one group created formulae as part of their model, while only one group used any form of a table (which they used in developing their model but chose not to display in their report.)

The children displayed various data transformation processes in developing their models, as indicated in Table 1. Group 1 created new, interval quantities (e.g., "Year 9 reads 4 books," "Year 9 reads 5 books etc.") and transformed these into other quantities by assigning value points (e.g., "= 1 point"), as can be seen in their documented account below.

Group 1's Model

Dear Margaret Scott
We have found a solution to the problem you have given us. Our information is below.

Year 9 reads 4 books = 1 point	8=4=2
Year 9 reads 5 books = 2 points	8=5=3
Year 9 reads 6 books = 3 points	8=6=4
Year 9 reads 7 books = 4 points	8=7=5
9=8=5	8=8=6
9=9=6	8=9=7
9=10=7	8=10=8

(The group continued the pattern for the grade 7 and grade 6 reading books.)

Group 2's Model

Group 2 quantified selected problem factors and transformed quantities into other quantities. In doing so, the children made use of weighting (multiplying a book's grade by 10), used interval quantities, assigned value points, and aggregated quantities.

Dear Mrs Scott
We have found a solution to your problem.
We think that the grade of the book times by 10 and then add the amount of pages to that. After doing this you should look at the book report grade and if it is between an A+ to an A- they get another 50 points, B+ to a B- 40 points, C+ to a C- 30 points, D+ to a D- 10 points and no points for an F. The three scores should be added together and that will be the score of the book. We chose this way because the person involved will easily get a high score. There for lifting their high esteem and they will be encouraged to read more which is the whole point of this activity. P.S the total score of all the books read should be added up in the end and that will be the total score of all of the books.

- The grade of the book x 10	A+ to A− = 50 points
- Plus the amount of pages in it	B+ to B− = 40 points
- Add the grade of the book report	C+ to C− = 30 points
	D+ to D− = 20 points
	E+ to E− = 10 points
	F = 0 points

Group 3's Model

Group 3 quantified only two factors, namely, a student's grade level and a book's reading level. The group also transformed quantities into other quantities by considering the difference between the student's grade level and the book's reading level, and then assigning points according to the extent of difference.

We have resolved the problem about the points issue. We have come up with a fair point system. We surround it by the level you read at. 2 points if you read at your level, 3 points above your level 2 levels higher. You get four points if it is 4 levels above your level. You get one point if it is one below your level. You get zero points if it is two levels below you. We think this is the easiest way to reward the children.
2 = year level
3 = above your level twice
4 = above your level 4 times
1 = below your level one times
0 = below your level two times

Group 4's Model

Group 4 created the most comprehensive model, taking into account all the problem factors. The group commenced the problem with a child commenting, "*Well, let's go over the possibilities like the number of books they read… Well, the number of books they read is up to them and what grades they do. So there should be like one point of order for each book they read – like, one bonus point. And depending on the grade, if they read a grade 4 book they get 8 points – like, you double it*." Another group member responded, "*But it depends. If they're in grade 10 and they read a grade 4 book, then they wouldn't really get any points for* *that*." In subsequently constructing their model the children weighted each book read (one bonus point awarded), assigned value points to the variety of books read (variety as determined by books from several grade levels having been read), and assigned points to the difficulty level of a book as it related to a student's grade level (informally toying with the idea of rate). It is interesting to note the "rules" and constraints on rules that the children included in their model.

Dear Miss Margaret Scott,
We have found a solution to your problem. But first we will tell you our strategy. For every book a student reads they would get 1 bonus point per book. Next if they read three different grades of books or more they would get 5 points. Eg. If a grade 4 girl/boy reads a grade 4, 5 and 6 book then he/she would get 5 points. We gave points for the difficulty of books by if you were in grade 6 and you read a grade 6 then you would get 3 points because you half the grade. Although there is a rule of you can only read two levels below your grade to receive points and as many levels above that you are capable of. If you were in grade 5 and you read a grade 4 book you would still get 4 (2?) *points because you are still halving it. But if you were in grade 9 and you read a grade 5 books you would not get any points. Because the book is too easy. We went by a code for the lengths of books: 50-70 = 3 points, 71-100 = 4 points, 101-170 = 5 points, 171-220 = 6 points and 221 and up = 7 points. We also made a code for the written reports as well which is: F=0, D=1, C- =2, C=3, C+ = 4, B- = 5, B=6, B+ = 7, A- = 8, A=9 and A+ = 10. we hope this helps you decide on how to give out points and awards during the summer.*

Group 5's Model

Group 5 considered the factors of grade level, book reading level, and report quality. Value points were awarded according to the level of a book read and the difference between a student's grade level and the book level. An informal notion of rate was evident in their assigning of points here.

Dear Margaret Scott
We have formulated a point system to determine results for your reading marathon. The system works on a point basis. If you read a book that is based for your grade level you receive 10 points although if you read a book higher than your grade level you receive 2 points for every grade level you read up and you 2 points go down for every grade level lower. If you get an F for the report you don't get bonus points but from every grade that goes up from F you receive one point.

DISCUSSION

Children's Models and Implications for Problem Design

This paper has addressed one way in which mathematics, science, and the arts can be brought together in meaningful learning experiences for primary school children. A model-based approach to students' mathematical learning, which takes children beyond their usual problem-solving experiences, provides rich opportunities for interdisciplinary experiences. Such an approach also engages children in future-oriented learning, both with respect to the more sophisticated mathematics they encounter and the collaborative team work they develop in dealing with complex situations. In contrast to children's regular mathematical activities, modelling places children at the centre of their own learning. That is, children are placed in authentic problem situations where they have to construct their own mathematics to solve the problem.

Modelling in both mathematics and science should not be confined to the secondary school years and beyond. As this paper and others have shown, primary school children are capable of engaging successfully with modelling problems that involve complex data systems. In working such problems children cycle through interpreting and re-interpreting the problem and the data sets, identify key problem factors, determine and apply quantification processes to transform the data, and document and support their actions in various representational formats. For example, in the Summer Reading problem the children debated which factors to include in their models and how to quantify them. In quantifying their data they assigned value points, used interval quantities, weighted some factors, aggregated quantities, and applied informal measures of rate. In so doing, the children created new quantities, transformed qualitative factors into quantities, transformed quantities into other quantities, and created formulae and lists to support their descriptions of their model construction.

Although it is difficult to make comparisons between the two studies, given the different student populations and prior modelling experiences, it is nevertheless a useful exercise to consider differences in the design of the two problems. These differences could account in part for the variation in diversity and sophistication of the models the children created on the two problems. The Creek Watch problem presented the children with a substantial amount of data (a Creek Watch Fact Sheet, an explanation of Pollution Index, a water bugs identification chart, a Moggill Creek data table, and the problem goal and background information). In dealing with the Moggill Creek data table (see Appendix A [4]), children need to consider the diversity and sensitivity of the macro-invertebrates at each site, the number and nature of fish present at each site (whether each fish is exotic or native), and the quantity of weeds found at each site and their impact on the water quality. The presence of chemicals at each site and their effect on the water quality also must be taken into account. In working with these data, children need to understand the pollution index and how (and whether) to apply it. Some groups ignored the PI while others calculated it for each site. The issue of how to deal with the remaining factors and how these might impact on the PI of the creek was a challenge for many children.

Although the children were able to create a variety of models to solve the problem, a few issues remain for attention. Why did some children simply aggregate all the fish and all the chemicals, ignoring their different features? Why didn't more children demonstrate an understanding that a balance of certain levels of chemicals in the water leads to optimum numbers and types of macro-invertebrates? Why didn't more models incorporate mathematisation processes and relationships between data? One possible explanation for these issues is that some children lacked an adequate understanding of the core scientific ideas, even though the teachers were asked to undertake preliminary work on these ideas. Another explanation lies in the nature and presentation of the data in the Moggill Creek data table. All of the data are presented in a quantitative format, in contrast to the Summer Reading problem where children have to quantify some qualitative factors (e.g., variety of books read). Perhaps the amount of additional scientific information given to the children in the Creek Watch problem was too excessive and required too much additional interpretation, drawing children's attention away from the important mathematical ideas inherent in the problem. The design of the problem itself could also be a contributing factor here. That is, it might be that the self-assessment principle was not applied adequately in the design of the problem—there might have been insufficient criteria for the children to assess their progress and determine whether their final model met the client's needs. The children were simply told to utilise "all the data collected over the year in five locations along the creek." A redesign of this problem could address this issue and also include some qualitative data to be quantified, as well as a restructuring of the data (e.g., fewer macro-invertebrates and a greater variety of chemicals and plants, some detrimental and others beneficial).

The Summer Reading problem, on the other hand, elicited more diverse and more sophisticated models than the Creek Watch problem. Although the difference in student populations is an obvious factor here, there are other reasons why this might be the case. The children were not presented with additional data to interpret and apply to problem solution as in the CreekWatch problem. Furthermore, the nature of the problem factors to be considered included both qualitative and quantitative data (e.g., variety of books versus number of books). The children thus had to come up with some way of determining how to quantify all of the factors, suggesting that this problem met the model construction principle better than the Creek Watch problem. It could also be that the Summer Reading problem more effectively met the self-assessment principle, that is, children were told explicitly the factors to consider in creating their model, not all of which were defined for them. Finally, although the problem was couched within a literary discipline, the problem did not draw upon as much additional disciplinary content as did the Creek Watch problem.

Interdisciplinary Projects with Modelling

As previously noted modelling problems can serve as unifying vehicles for the primary curriculum, bringing together key ideas from several disciplines. For example, in the Creek Watch problem, core concepts are drawn not only from mathematics, but also from science and from studies of society and the environment. Such concepts include the dynamic nature of environments and how living and non-living components interact, the ways in which living organisms depend on others and the environment for survival, and how the activities of people can change the balance of nature. In providing a literary context, the Summer Reading problem draws children's attention to the features of an array of books and encourages them to consider how book length, difficulty level, variety of books read, and quality of book reports could determine points awarded to readers.

Because of their interdisciplinary nature, modelling problems provide a rich platform for student research projects. There are numerous opportunities for generating such projects within the regular curriculum. One such opportunity, which I utilised recently in three fifth-grade classes, engaged children in an investigation of their country's settlement. The children were commencing a study of the arrival of the First Fleet on the east coast of Australia in 1787. A modelling problem was created to tie in with their study of this settlement. As part of the problem, the children explored the reasons behind the first settlement, the composition of the First Fleet (comprising 11 ships), the difficulties faced by the Fleet en route to their destination, the types of supplies on board the ships, and the various conditions required for the settlement of a new colony. The problem text explained that, on his return from Australia to the UK in 1770, Captain James Cook reported that Botany Bay had lush pastures and well-watered and fertile ground suitable for crops and for the grazing of cattle. But when Captain Phillip (commander of the First Fleet) arrived in Botany Bay in January 1788, he thought it was unsuitable for the new settlement. Captain Phillip headed north in search of a better place for settlement. The children's task was as follows:

Where to locate the first settlement was a difficult decision to make for Captain Phillip as there were so many factors to consider. If you could turn a time machine back to 1788, how would you advise Captain Phillip? Was Botany Bay a poor choice or not? Early settlements occurred in Sydney Cove Port Jackson, at Rose Hill along the Parramatta River, on Norfolk Island, Port Hacking, and in Botany Bay. Which of these five sites would have been Captain Phillip's best choice? Your job is to create a system or model that could be used to help decide where it was best to anchor their boats and settle. Use the data given in the table and *the list of provisions on board to determine which location was best for settlement. Whilst Captain Phillip was the first commander to settle in Australia many more ships were planning to make the journey and settle on the shores of Australia. Your system or model should be able to assist future settlers make informed decisions about where to locate their townships.*

As for the Creek Watch problem, the First Fleet problem elicits similar core ideas from science and from environmental and societal studies; these ideas serve as springboards for new investigations of other early settlements in Australia. The problem can also lead nicely into a more in-depth study of the interrelationship between ecological systems and economies, and a consideration of ways to promote and attain ecologically sustainable development.

In another study, fourth- and fifth-grade classes worked two modelling problems exploring Australia's cyclones as part of their study of natural disasters (English, Fox, & Watters, 2005). In the first problem the children explored data pertaining to cyclone categories and how these are determined, the wind speeds of cyclones, their impact on the environment (e.g., landslides, flooding, destruction of buildings, loss of power etc), and also the locations and severity of cyclones in Queensland (the children's home state) in the last 12 years. The children were to take on the role of assisting a Project Resort Development Committee to assess possible locations for building new holiday destinations in Queensland. Using the data supplied, the children were to develop a model that would determine which two locations the resort development company should avoid. The follow-up problem required children to do their own research on cyclone activity in another Australian state, namely, Western Australia. The children researched and identified data that they considered important in determining suitable sites for new resorts. The models the children developed for the first problem were then applied to this second example, with some groups of children making improvements to their original models.

CONCLUDING POINTS

Designing appropriate interdisciplinary modelling experiences is not an easy task for teachers or researchers. Numerous cycles of creating, testing, re-building, and refining a problem are usually required. The instructional design principles cited previously provide an effective guide in creating these modelling problems. Consideration needs to be given not only to the mathematical ideas to be embedded in the problem but also to the other discipline content. Determining an appropriate balance of interdisciplinary ideas is an important aspect of the problem design and will be governed in part by the nature of the disciplines and the age of the children involved. Also of importance is the need to capitalize on the myriad opportunities across the curriculum for children to undertake interdisciplinary research projects involving modelling problems. Such projects both develop and extend multi-disciplinary learnings and enable children to review, adapt, and apply their model creations.

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APPENDICES

Appendix A (1) The Creek Watch Problem[1]

INDOOROOPILLY TIMES

State department and local school kids work together to keep watch over Moggill Creek

Increased urban development in the Western Suburbs of Brisbane is threatening the quality of the local environment. In particular, runoff is impacting on water quality and visible signs such as rising salinity and blue-green algal blooms are becoming more prevalent.

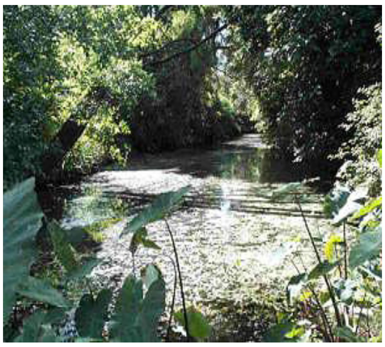
Since 1995, scientists from the Queensland Environmental Protection agency (EPA) and Brisbane City Council (BCC) have monitored the water quality in many creeks in South East Queensland.

In 2004 Year 5 students from Indooroopilly State School (ISS) assisted these authorities to collect important data about Moggill Creek. The Creek flows into the Brisbane River in the Western Suburbs. The members of the class collected water samples from the creek to test for chemicals. Jack Simpson, a student from ISS said, “The class also took samples of different species of fish and macro-invertebrates. I loved looking at the macro-invertebrates the best, they are great water bugs.”

Mrs. Jones from the EPA said the students’ assistance had been invaluable and the information they provided helpful. “By monitoring the waterways we can gain a picture of catchment health. Monitoring over time can provide information on the state of the catchment which can assist with the maintenance and rehabilitation of our waterways.”

When deciding how healthy Moggill Creek is, many factors have to be investigated and recorded over a period of time. A healthy river has high amounts of dissolved oxygen and low amounts of phosphorous and nitrogen. It also has relatively low salinity. “We have to combine all of these different chemical and biological measurements to come up with an indication” said Mrs. Jones.

Below: A stretch of Moggill Creek where students from ISS collected valuable data for the study.



Apart from the water quality components, other factors help to determine how healthy the creek is. “Putting the chemical data together with the information that we gather about the fish and macro-invertebrates tells us how healthy the river is at that particular moment” said Mrs. Jones.

[1] *The activities displayed in each appendix were developed by Lyn English and James Watters with assistance from Jo Macri*

Mrs Jones said “We know that certain macro-invertebrates are highly sensitive to pollution whereas others have low sensitivity and survive in polluted water. By counting the different types of macro-invertebrates and knowing their sensitivity we can work out the pollution index of a creek.”

The river is considered clean and healthy when the river has lots of different species of water bugs, particularly highly pollution sensitive macro-invertebrates.

Obtaining samples is a time-consuming job. Having students gather data from different sites along Moggill Creek helps the department identify which areas are healthy and which sites need the most help in getting cleaned up.

While the children are supporting the local community through their investigations, they are also learning about river ecology and why it is important to keep pollution down in waterways.

Left: A creek highly polluted showing an algal bloom

READINESS QUESTIONS

1. What are macro-invertebrates? (draw a picture of an example)
2. Why does the EPA want students to help collect data for them about the conditions of Moggill Creek?
3. When scientists want information on “dissolved oxygen, phosphorous and nitrogen” what are they looking for?
4. What levels of dissolved oxygen, phosphorus, nitrogen, and total salinity does a healthy river have?
5. Why is it important to know both the number of organisms and the amount of each species that a creek site has?
6. Why is the students’ involvement in the creek monitoring task a good idea?
7. What is the pollution index of a creek and how do you calculate it?

MODELING ACTIVITY

Jack Simpson’s class is presenting their information at a community meeting where other interested groups are presenting their conclusions. The meeting organisers are offering a prize for the group that develops the best system that describes the most important criteria in establishing the total water quality of a creek.

Jack’s group needs your help to construct a model or set of guidelines that indicates the health of the creek. Your system should make use of all the data collected over the year in five locations along the creek. These data are shown in the Table attached. Jack’s class started collecting near the source of the Creek (Site E) and took samples all the way to the mouth of the Creek where it entered the Brisbane River (Site A)

Write a letter to Jack’s group that describes how you developed your system so that it can be used by others in determining the health of any creek.

Appendix A(2) Explanation of Pollution Index

Pollution index[2]

There are a number of different things that can pollute water and consequently affect the distribution of macro-invertebrates. Pollutants include domestic waste and animal wastes (e.g. from paddocks, dairies, horse stables and yards.) These wastes can contribute to the development of toxins, bacteria, and viruses. They enter water courses through run off, or seep in through ground water. The quality of the water can be determined by calculating a pollution index.

Pollution index

Macro-invertebrates can be divided into three groups according to how sensitive they are to pollution and assigned a number related to their group:

Sensitive	5-10
Tolerant	3-4
Very Tolerant	1-2

Each animal has a number or score next to it in the water bug table.

When you have completed the collection and identification, add the numbers assigned to each animal. For the index, only count each type of animal once. Clean water will have a high total score because it can support a lot of pollution sensitive bugs.

High abundance of only a few species might indicate poorer conditions.

[2] Water and Rivers Commission of Western Australia: http://www.wrc.wa.gov.au/public/waterfacts/2_macro/water_condition.html

Appendix A(2) Fact Sheet for the Water Watch problem

Fact Sheet

Water quality is determined by measuring the presence of macro-invertebrates in the water, the presence of fish, algae and weeds, and the concentration or amounts of various chemicals.

M**acroinvertebrates** are animals without backbones that are big enough to see with the naked eye. Examples include most aquatic insects, snails and crustaceans such as yabbies. Some macro-invertebrates tolerate polluted water well while others are very sensitive to pollution. Thus the diversity of macro-invertebrates and types of species are good indicators of the quality of water in a creek. By counting the different species of macro-invertebrates Scientists have devised a pollution index to describe the state of pollution of the creek.

F**ish**: There are many hundreds of species of native Australian freshwater fish. Some species have been around for over 60 million years and have become adapted to the unique Australian conditions. However many species are now threatened by the introduction of Exotic or Alien fish. ***Exotic species*** are those species brought into Australia from a foreign country mostly as aquarium fish. These have been released into the wild from home fish tanks. Common exotic species are Guppies, Swordtails and Platys. Although these species are small fish, they compete for available food resources to the disadvantage of some native fish. The Mosquito Fish (*Gambusia holbrookii*) was introduced in the past in the hope of providing insect pest control services but it has proven itself to be a voracious controller of tadpoles, too. The health of our creeks is indicated by the abundance and diversity of native fish.

P**lants:** Various plants grow in and around Queensland creeks and billabongs. Plants are important as they produce oxygen which dissolves in the water and helps fish and animals survive. Bulrushes and reeds are common indigenous plants which have root systems whereas water lilies are floating plants. Unfortunately there are many introduced exotic species. Exotic species of plants have caused havoc in some areas. Waterlettuce, alligator weed, fanwort (Camomba), water hyacinths, and salvinia are particular unwelcome weeds because they grow rapidly and choke out native plants and fishes. Chinese elm and Camphor Laurel are also considered as weeds. Others such as cape blue water lily are useful as they contribute to the health of the water way by stimulating oxygen production.

Algae: Blue green algae are an important part of a healthy creek. Although quite rare, some species have the potential to produce toxins in low quantities which kill fish and can harm animals and swimmers. In normal conditions these plants and algae exist in low numbers in the waterways at no detriment to the environment or to human health. However under favourable conditions native plant species can be out competed. Weeds and algal colonies can over grow natural habitats. In large numbers this harms waterways attacking native aquatic plants and animals.

Chemicals: The presence of various chemicals and matter is an important indicator of water quality. Pure water is H_2O. Chemicals from the soil, such as sodium chloride, contribute to salinity. Thus sea water is described as salty because it contains lots of common salt or sodium chloride. Two important chemicals are phosphorus and nitrogen. Compounds containing these chemicals often contribute to the growth of phytoplankton which are microscopic animals that form algae blooms. After storms, saltwater swimming pools either overflow or their owners drain them to lower water level. If this coincides with reduced stream flow the salinity can rise markedly.

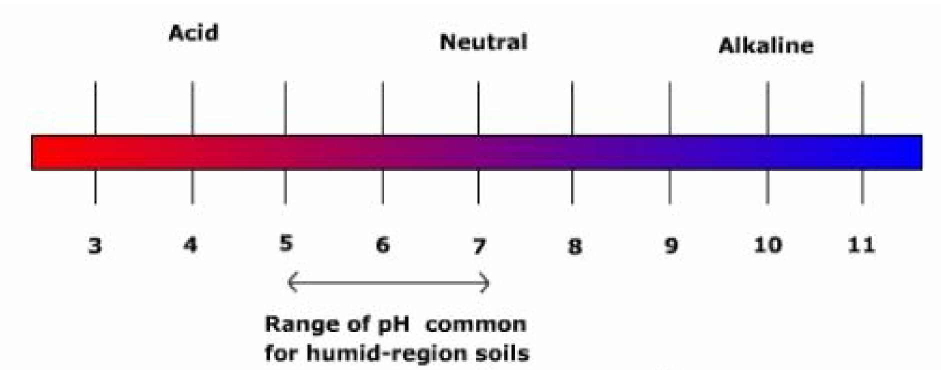
Phosphates are common polluting chemicals found in fertilizers. So run off from farms and gardens add excess amounts of phosphorus to the water. However, phosphates also come from the natural weathering of rocks and the decomposition of organic material. Sewerage is rich in phosphorus. The higher the level of phosphate the poorer the quality of water.

Nitrogen containing substances also occur naturally in water however human activity contributes excess nitrogen to our waterways. Nitrogen is usually measured as nitrate and occurs in nitrates, nitrites and ammonia. Nitrates occur in fertilizer and hence run off containing nitrates from farms and gardens leads to a proliferation of plant life which competes with other life. The higher the level of nitrogen the poorer the quality of water.

Oxygen: Oxygen is essential for all animals. Oxygen is what we breathe from the air. Dissolved oxygen is probably the most critical water quality variable in freshwater creeks. Fish need oxygen. Oxygen levels in creek systems depend on water temperature, salinity, and the amount of aquatic vegetation and number of aquatic animals in the creek. It also depends on the flow rate of the creek and the extent to which the water gets churned up in passing over rocks. When there is too much growth of microscopic algae (phytoplankton) and other oxygen consuming micro-organisms larger species of animals die off. High levels of oxygen are important for good quality water.

Turbidity: The clarity of water is also a useful indicator of quality although being cloudy does not necessarily mean the water is polluted. Muddiness or *turbidity* could be caused by silt or dirt but it could also be caused by bacteria and algae. Turbidity prevents light from getting to plants to help them grow. High levels of turbidity are detrimental.

Acid: pH is the measure of the amount of acid in water. pH describes the acidity of water and low pH means highly acidic. Vinegar and lemon juice are acidic and have low pH whereas soap and washing powder are described as alkaline – the opposite of acidic – and have high pH. The strange thing is that the more acid the lower the pH. Normal pH is about 7.0 and most organisms can survive between pH6.5 and pH9.0. Acid gets into creeks through increased pollution from burning fossil fuels. Alkaline materials are often leached from rocks that the creek passes over.

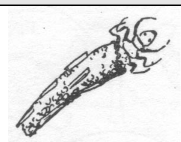
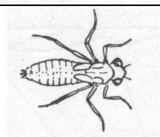
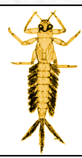
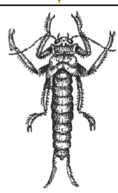
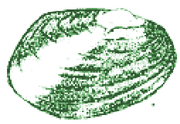
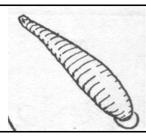


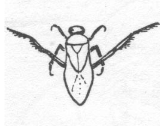
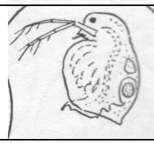
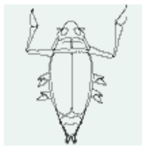
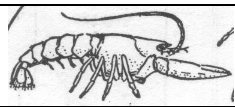
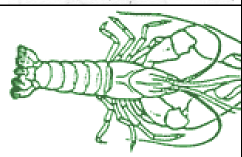
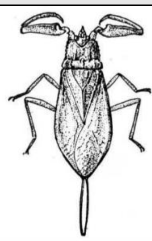
Weeds: Invasive weed infestations crowd out and smother native plants and hence have an adverse impact on the health of the ecosystem. Native butterflies, birds and other insects rely on native plants. These exotic invasive weed are distributed by wind, birds, or as the result of dumping of garden rubbish. Poor quality water ways have dense growths of weeds.

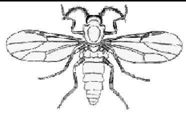
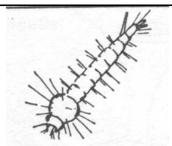
Appendix A(3) Water Bugs Identification Chart

Water Bugs Identification Chart

The health of a stream is given in terms of the diversity of species and the relative tolerance level of various species.

Species	Size	Features	Drawing	Score
		High Sensitivity		
Caddisfly larvae	Up to 20mm	They live in a wide range of environments from fast flowing streams to freshwater ponds. Their soft bodies are usually covered in a protective silky case		6
Dragonfly nymph	18-50mm	Dragonfly Nymphs are short and chunky with wing pads and internal gills. Their six legs are all located near the head		6
Mayfly Nymph	10-20 mm	These are only found in very clean water containing lots of oxygen. They absorb oxygen from the water through their gills.		7
Stonefly nymph	Up to 50 mm	They have three segmented tarsi and long antennae. Require lots of oxygen		8
Water mite	5mm	Adults are free-swimming or crawling. Most common in heavily vegetated wetlands – often parasitic on other insects		5
		Medium Sensivity		
Fairy shrimp	10-30 mm	Related to brine shrimp, copepods and Daphnia		3
Freshwater mussel		Mussels are soft bodied animals enclosed in two hinged shells		3
Leech	3-15mm	Leeches are segmented worms with a sucker on one or both ends. They are found free swimming in water as well as on plants or on the bottom.		3

Planarian	2-5mm	These are free living flat worms which possess a remarkable ability to regenerate their lost body parts		3
Pond snail	10-20mm	Aquatic snails are similar in form to land snails but smaller		3
Water boatman		Water boatmen and water striders are bugs. These tend to be shield shaped when viewed from above. Their soft front wings are folded and overlap to leave a small triangle on their back.		3
Water strider		Water boatmen and water striders are bugs. These tend to be shield shaped when viewed from above. Their soft front wings are folded and overlap to leave a small triangle on their back.		3
Water tiger beetle		Beetle larvae are segmented, have three distinct pairs of legs. They are usually active with large mouth parts.		3
Waterflea	1 mm	Also known as daphnia these are related to crabs and prawns.		3
Whirligig beetle	3-35mm	They congregate in large numbers and scurry about the water surface in a random pattern. Shiny to dull black, often with a bronzy sheen.		3
Shrimp		Shrimp are small crustations that look similar to prawns.		5
Yabby		Freshwater crayfish that are commonly found in ponds and streams.		5
Low sensitivity (Tolerant)				
Water Scorpion	30mm	Noted for the first pair of legs which are modified into prehensile organs for grasping prey. They are carnivorous and feed on smaller insects. The prey is held securely between their first pair of legs while the water scorpion sucks up its body fluids. Tends to be found on the muddy bottom of creeks.		2

Midge		These are small pesky biting insects as adults but are slender worm-like creatures, sometimes red, with no legs.	 	2
Mosquito lavae		These animals twist and squirm just below the surface of the water. The larvae look like hairy maggots with siphons.		2

APPENDIX B(1)

SUMMER READING PROBLEM

Brisbane – While a long hot summer may be ahead of us, the Brisbane City Council Library (BCCL) is offering a chance for patrons to stay cool this year.

The annual "Reading is Cool" summer reading program will officially start at noon, June 1, in the Indooroopilly Room. Students from St. Peters will receive a free library card that will let them participate in the program.

Students can choose from an approved collection of books that the library has placed on reserve. The books have been classified by grade level (according to difficulty of the book), to help the students choose which books to read. However, students may read any of the books, regardless of their current grade level.

St. Peters students who participate will have the chance to not only earn prizes from the library, but also prizes from their school. The St. Peters School and the BCCL have teamed up to provide prizes for overall winners and classroom winners.

Some prizes that the students can win, based on a point system, include bookmarks, books, T-shirts, hats, meals from local restaurants, and compact discs. Classroom winners will also be eligible for a chance to win a $300 savings account.

To register, simply stop by the Brisbane City Council Library, 318 Moggill Rd, between 9 a.m. and 9 p.m. The contest ends Aug. 12, with the final day to collect prizes Aug. 25.

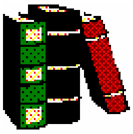
The library is accepting suggestions for this year's reading contest. To give your input, please send a letter to Lynn, the Indooroopilly reading coordinator. All suggestions must be received by May 1.

Ready to Go: The books are all shelved at the Brisbane City Council Library in Indooroopilly. Participating students can choose from over 250 books for extra bonus points in this year's summer reading program.

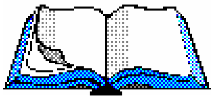
Summer Reading Program Readiness Questions

Answer the following questions using the journal article and the tables given below.

1. Drew read The Tell-Tale Heart and Roll of Thunder, Hear My Cry. Should he receive the same number of points for each book? Why or why not?



2. If a sixth grader and a ninth grader both read A Tale of Two Cities, should they both earn the same number of points? Why or why not?



3. If Shelly reads Jurassic Park and Much Ado About Nothing, should she get the same number of points for each?

EXAMPLES OF APPROVED BOOKS

TITLE	AUTHOR	READING LEVEL (BY GRADE)	PAGES
Sarah, Plain and Tall	Patricia MacLachlan	4	58
Are You There God? It's Me Margaret.	Judy Blume	4	149
Awesome Athletes	Multiple Authors	5	288
Encyclopedia Brown and the Case of Pablo's Nose	Donald J. Sobol	5	80
Get Real (Sweet Valley Jr. High, No.1)	Francine Pascal, Jamie Suzanne	6	144
Roll of Thunder, Hear My Cry	Mildred Taylor	6	276
The Tell-Tale Heart	Edgar Allan Poe	6	64
Little Women	Louisa Mae Alcott	7	388
The Scarlet Letter	Nathaniel Hawthorne	7	202
Aftershock (Sweet Valley High)	Kate Williams, Francine Pascal	8	208
Jurassic Park	Michael Crichton	8	400
A Tale of Two Cities	Charles Dickens	9	384
Lord of the Flies	William Golding	9	184
Much Ado About Nothing	William Shakespeare	10	75

TITLE	*BRIEF DESCRIPTION OF BOOK*
Sarah, Plain and Tall	When their father invites a mail-order bride to come and live with them in their prairie home, Caleb and Anna are captivated by their new mother and hope that she will stay.
Are You There God? It's Me Margaret	*Faced with the difficulties of growing up and choosing a religion, a twelve-year-old girl talks over her problems with her own private God.*
Awesome Athletes	*Sports Illustrated for Kids*
Encyclopedia Brown and The Case of Pablo's Nose	*America's Sherlock Holmes in sneakers continues his war on crime in ten more cases.*
Get Real (Sweet Valley Jr. High, No. 1)	*Describes the trials and tribulations of twins that moved to a new junior high school.*
Roll of Thunder, *Hear My Cry*	*A black family living in the South during the 1930s is faced with prejudice and discrimination that its children do not understand.*
The Tell-Tale Heart	*The murder of an old man is revealed by the continuous beating of his heart.*
Little Women	*A story of family, of hope, of dreams, and of growing up as four devoted sisters search for romance and find maturity in Civil-War era 19th century New England.*
The Scarlet Letter	*Hawthorne's masterpiece about Hester Prynne, hapless victim of sin, guilt and hypocrisy in Puritan New England.*
Aftershock	*Twins deal with the pain and shock of an earthquake.*
Jurassic Park	*A modern-day scientist brings to life a horde of prehistoric animals and dinosaurs.*
A Tale of Two Cities	*A highly charged examination of human suffering and human sacrifice, private experience and public history, during the French Revolution.*
Lord of the Flies	*The classic tale of a group of English school-boys who are left stranded on an unpopulated island.*
Much Ado About Nothing	*Shakespeare comedy.*

Summer Reading Problem

Information: The Brisbane City Council Library and St. Peters School are sponsoring a summer reading program. Students in grades 6-9 will read books and prepare written reports about each book to collect points and win prizes. The winner in each class will be the student who has earned the most reading points. The overall winner will be the student that earns the most points. A collection of approved books has already been selected and put on reserve. See the previous page for a sample of this collection.

Students who enroll in the program often read between ten and twenty books over the summer. The contest committee is trying to figure out a fair way to assign points to each student. Margaret Scott, the program director, said, "Whatever procedure is used, we want to take into account: (a) the number of books, (b) the variety of the books, (c) the difficulty of the books, (d) the lengths of the books, and (e) the quality of the written reports.

Note: The students are given grades of A+, A, A-, B+, B, B-, C+, C, C-, D, or F for the quality of their written reports.

Your Mission . . .

Write a letter to Margaret Scott explaining how to assign points to each student for all of the books that the student reads and writes about during the summer reading program.

Table 1 Representation Formats, Problem Factors, and Mathematical Operations used by each of Five Student Groups in Model Development

Group	Representation Format					Problem Factors						Mathematical Operations				
	Table	Text	List	Computation	Formulae	No. of books	Variety	Length	Student's grade level	Book's reading level	Report's quality	Assigning value points	Using interval quantities	Using weighting	Aggregating quantities	Informal rate
1		✔	✔		✔	✔			✔			✔	✔			
2		✔	✔	✔	✔			✔		✔	✔	✔	✔	✔	✔	
3		✔	✔		✔				✔	✔		✔				
4		✔			✔	✔	✔	✔	✔	✔	✔	✔	✔	✔		✔
5	✔	✔							✔	✔	✔	✔	✔			✔